

**THE GLOBAL FOREST SECTOR MODEL EFI-GTM  
– THE MODEL STRUCTURE**

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**ABSTRACT**

This article presents the structure and the mathematical specification of the EFI-GTM model at the European Forest Institute. The EFI-GTM is a multi-regional and multi-periodic forest sector model that integrates forestry, forest industries, final forest industry product demand and international trade in forest products. The model includes at present 61 regions covering the whole world, but the special focus is in Europe. The products modeled currently include 6 wood categories, 26 forest industry products and 4 recycled paper grades.

The model calculates periodical production, consumption, import and export quantities and product prices for the forest sector products as well as periodical capacity investments of the forest industry for each region. The dynamic changes from year to year are modeled by recursive programming. In each period, the producers are assumed to maximize their profits subject to the production possibility set, while consumers are assumed to maximize their welfare subject to the consumption possibility set. Both producers and consumers are modeled as price takers, i.e., the model assumes competitive markets. In this report, we discuss in more detail how these assumptions are cast into the model.

## 1. INTRODUCTION

This paper describes the structure of the global forest sector model EFI-GTM at the European Forest Institute. Some simplified, one may say that the main function of the EFI-GTM is to provide consistent analysis of how and by how much production, consumption, imports, exports, and prices of roundwood and forest industry products might change over time as a response to changes in external factors like economic growth, forest biodiversity protection, energy prices, trade regulations, transport costs, exchange rates, forest growth, and consumer preferences. Because the model provides price projections, it gives relevant information also for private investment concerns.

The model consists of a group of competing economies that are trading forest sector commodities whenever the trade increases their economic welfare. In each economy, consumers are assumed to maximize their utility and producers are assumed to maximize their profits under perfect competition. For each region we define demand functions for the final products (mechanical forest industry products, paper and paperboard), supply functions for waste paper and timber, as well as a set of technologies for producing intermediate (pulp, chips) and final products.

The EFI-GTM is a *partial* equilibrium model, because it includes forestry and the forest industry, but the existence of the other sectors in the economy are only accounted for indirectly, via exogenous specification of demand functions and prices of exogenous production factors such as labor, energy and capital, and via predictions on possible technological change and economic growth. The model is *spatial* because it gives, for each period and region, an equilibrium solution which includes trade between all regions in the model.

The theoretical basis for the model is that of spatial equilibrium in competitive markets as first solved by Samuelson (1952) for several commodities. The competitive market equilibrium is found via maximizing the sum of producer and consumer surpluses net of transportation costs subject to material balance, trade, and capacity constraints.

The model optimization is static, but the model is multi-periodic. The model is solved for one period at time and the solution is used to update the timber supply specifications and the

existing production capacities for the subsequent period. Also, the given periodical data on the exogenous sector factors are then updated. Thereafter, a new equilibrium is computed subject to the new demand and supply conditions. As such, the dynamic changes from year to year are modeled by recursive programming, meaning that the long run spatial market equilibrium problem is broken up into a sequence of short run problems, one for each year. Hence, in the model it is assumed that the decision makers in the economy have imperfect foresight.

The use of forest sector models that integrate dynamics of forest resources, timber supply, forest industry, and forest product market demand has a long tradition. Among the most well known of such models are the Timber Assessment Market Model/TAMM (Adams and Haynes 1980), the PELPS model developed at the University of Wisconsin (Buongiorno and Gilles 1984, Zhang et al. 1993) and further modified for the Global Forest Product Model (GFPM) (Zhu et al. 1998, Buongiorno et al. 2003), and the Global Trade Model (GTM) developed at the International Institute for Applied Systems Analysis (IIASA) in the late 1980s (Kallio et al. 1987).

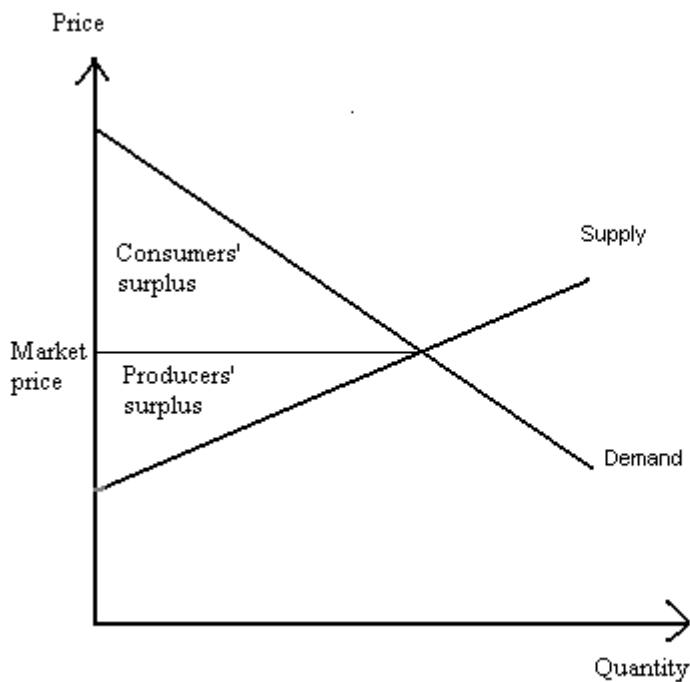
The multiregional partial equilibrium model GTM has been tailored by various research institutes to best suit their research orientation. The EFI-GTM model described here is largely based on the same structure as the original GTM described in Kallio et al. (1987). However, the model is more disaggregated regarding regions and production technologies. The special focus of the model is in Europe. Besides EFI-GTM, other global applications of the GTM include the CGTM-model applied at the University of Washington (e.g., Cardellicchio et al. 1989, Perez-Garcia 1993). In addition, there are or have been several regional applications of the GTM, e.g., ATM for Austria (Kornai and Schwarzbauer 1987), SF-GTM for Finland and Western Europe (Ronnala 1995), and NTM for Norway (Trømborg and Solberg 1995, Bolkesjø and Solberg 2003).

Before going to the particular details of the EFI-GTM in Section 3, we describe the structure of a general partial equilibrium model in the next section. As much of the details in the model structure have not changed since the GTM model, the discussion in these sections is, to some extent, adopted from Chapters 19–25 of Kallio et al. (1987). In particular, we capitalize on the framework by Salo and Kallio (1987). The data use of the model is documented in more detail in Moiseyev et al. (2004), but the overview of the current data sources of the model are given in Table A1 of the Appendix.

## 2. GENERAL STRUCTURE OF A PARTIAL EQUILIBRIUM MODEL

We shall start by clarifying a general structure of any partial equilibrium model by considering several levels of hierarchy. At the lowest level, the sectoral agents – producers, consumers, and trade agents – maximize their welfare given specific constraints. Together they form a regional economy, where the objectives and the constraints of the individual agents are aggregated and a further constraint is added for each product in each region: the sum of consumption in the region and exports from the region to other regions must equal the sum of production in the region and imports to the region from the other regions. The global model links the regional modules together through inter-regional trade.

Let us now briefly look into problems at each level and how they are aggregated into a single mathematical programming model, which is used to solve for competitive equilibria in all the markets simultaneously.



**Figure 1.** Consumers' and producers' surplus in market equilibrium.

## CONSUMERS

Assume that consumers attempt to maximize their welfare, which depends on the consumption of the final products. With the separable demand functions, this welfare is greatest when consumers' surplus, defined as the area below the demand curve and above the market price, as illustrated in Figure 1, is maximized for each product.

The consuming sector is assumed to consist of numerous agents that take prices as given. In a given region  $i$ , let  $q^i = (q_k^i)$  be a vector of the consumed quantities, and  $P_k^i(q_k)$  be the inverse demand function for product  $k$ . Assume that  $P_k^i(q_k)$  is differentiable and non-increasing. Let  $\mathbf{p}^i = (\mathbf{p}_k^i)$  be a vector of product prices, and let  $Q^i$  denote the consumption possibility set, which is assumed to be closed, convex, and non-empty. Then the consuming sector's problem is

$$\text{Max}_{q^i} \quad \sum_k \int_0^{q_k^i} P_k^i(q_k) dq_k - \mathbf{p}^i q^i \quad (1)$$

s.t.

$$q^i \in Q^i. \quad (2)$$

## PRODUCERS

Assume that producers (e.g., timber growers and forest industry firms) of a given region  $i$  maximize their profits, defined as producer's surplus. Let  $z^i = (z_k^i)$  be a vector of net output volumes for products  $k$  in region  $i$ , let  $C_k^i(z_k)$  be the marginal cost function for product  $k$ , and let  $V^i$  be a closed, convex, and non-empty production possibility set. Under competitive markets, the producers take product prices  $\mathbf{p}_k^i$  as given. Then the producers' problem is the following:



$$\text{Max}_{z^i} \quad \mathbf{p}^i z^i - \sum_k \int_0^{z_k^i} C_k^i(z_k) dz_k \quad (3)$$

s.t.

$$z^i \in V^i. \quad (4)$$

### TRADE AGENTS

Trade can be considered as a separable activity carried out by trade agents. These agents can, of course, be the initial producers. To maximize the profit from trade, exporters buy goods at the domestic price, pay for the transportation, and sell at the price of the importing region. Similarly, importers buy at import prices and aim to make profits by selling at the domestic prices. The problem faced by a trade agent operating in region  $i$  is the following:

$$\text{Max}_{e_k^{ij}, e_k^{ji}} \quad \sum_{jk} [(\mathbf{p}_k^j - \mathbf{p}_k^i - D_k^{ij}) e_k^{ij} + (\mathbf{p}_k^i - \mathbf{p}_k^j - D_k^{ji}) e_k^{ji}] \quad (5)$$

where  $\mathbf{p}_k^i$  is the price for product  $k$  in region  $i$ ;  $\mathbf{p}_k^j$  is the price for product  $k$  in region  $j$ ;  $e_k^{ij}$  are exports for product  $k$  from region  $i$  to  $j$ ;  $e_k^{ji}$  are imports from region  $j$  to  $i$ ; and  $D_k^{ij}$  is the transportation cost for a unit of product  $k$  from region  $i$  to  $j$ .

### REGIONAL MODELS

The objective function for region  $i$  is specified by adding up the agents' objectives (1), (3), and (5), while the feasible set for the regional problem consists of the constraints (2) and (4) of the agents and the material balance equations

$$q_k^i + \sum_j e_k^{ij} = z_k^i + \sum_j e_k^{ji}, \quad \forall k. \quad (6)$$

As equations (6) hold in equilibrium, the regional objective function can be reduced to a form where the vector  $\mathbf{p}^i$  of domestic prices vanishes. The problem related to economy  $i$  will then be

$$\text{Max} \quad \sum_k \int_0^{q_k^i} P_k^i(q_k) dq_k - \int_0^{z_k^i} C_k^i(z_k) dz_k + \sum_{jk} [(\mathbf{p}_k^j - D_k^{ij}) e_k^{ij} - (\mathbf{p}_k^j + D_k^{ji}) e_k^{ji}] \quad (7)$$

subject to constraints (2), (4) and (6).

### THE GLOBAL MODEL

The eventual global model consists of the constraints (2), (4) and (6) for all regions  $i$  and the sum of objective functions (7) of all the regions. As the imports of product  $k$  to region  $i$  from region  $j$  equal the respective exports from  $j$  to  $i$ , the import variables  $e_k^{ji}$  match the export variables  $e_k^{ij}$ .

Thereby, the global problem is to find  $q^i$ ,  $z^i$ , and  $e_k^{ij}$  to maximize

$$\sum_{ik} \left[ \int_0^{q_k^i} P_k^i(q_k^i) dq_k^i - \int_0^{z_k^i} C_k^i(z_k^i) dz_k^i \right] - \sum_{ijk} D_k^{ij} e_k^{ij} \quad (8)$$

subject to constraints (2), (4) and (6) for all  $i$ .

The optimality conditions for the problem above equal the equilibrium conditions for regional competitive markets – as proved first by Samuelson (1952).

### 3. DETAILED SET-UP OF EFI-GTM

Let us now discuss the more detailed specification of the EFI-GTM. We shall start with the different sectors in the model: consumers, forestry and the forest industry. Thereafter we present the eventual NLP (nonlinear programming) program for the multi-regional model. Because the model is solved by calculating the results for each period separately, subscripts  $t$  referring to time periods have been left out, whenever they have not been considered necessary for understanding the presentation.

#### CONSUMER SECTOR

Consumers of the final products  $f$  are represented via demand functions, which are specified for each product and each region  $i$  employing the data on the reference (observed) demand,  $\hat{q}_f^i$ , reference price,  $\hat{p}_f^i$ , and the econometrically estimated price elasticity of the demand,  $g_f^i$ , in a region. For solving the model, we use a linearization of a (constant elasticity) demand function. We assume that the observed reference values of price and quantity are taken from the demand function. The parameters of the demand function are specified so that at the reference point, the demand elasticity equals  $g_f^i$ . Consequently, the following linear demand function arises:

$$q_f^i = (1 - g_f^i) \hat{q}_f^i + \left( \frac{g_f^i \hat{q}_f^i}{\hat{p}_f^i} \right) p_f^i \quad . \quad (9)$$

where  $q_f^i$  is a quantity of product  $f$  demanded at price  $p_f^i$  in region  $i$ . In solving the model, the inverse  $P_f^i(q_f)$  of Eq. (9) is used.

The reference demand  $\hat{q}_f^i$  is updated in each period after the base year to account for the exogenously given forecasts of the GDP growth,  $b_t^i$ , in period  $t$ , and the econometrically estimated elasticities of the demand of product  $f$  with respect to the GDP,  $e_f^i$ , in region  $i$ . The updated reference demand  $\hat{q}_f^i$  in any period  $t$  after the base year is

$$\widehat{q}_{f,t}^i = (1 + b_t^i e_f^i) \widehat{q}_{f,t-1}^i. \quad (10)$$

Respectively, Eq. (9) is redefined based on (10) for each period after the base year.

## ROUNDWOOD SUPPLY AND FORESTRY

Through the study, we assume that timber markets are perfectly competitive and that timber growers maximize their income for each period separately. Marginal costs are assumed to be an increasing function of the harvested volume.

When defining the regional timber supply functions for regions  $i$ , the starting point was the observed harvesting quantities,  $\widehat{h}_w^i$ , for different timber categories  $w$ , observed timber prices,  $\widehat{p}_w^i$ , respectively, and econometric estimates for the inverse supply elasticities of timber,  $b_w^i$ . The inverse supply function for timber category  $w$  in region  $i$  is specified as:

$$p_w^i = d_w^i + a_w^i h_w^i b_w^i, \quad (11)$$

where  $p_w^i$  is a timber price per cubic meter,  $d_w^i$  is an exogenous cost component in the timber price that does not depend on the harvest level,  $h_w^i$  is a harvest level and  $a_w^i$  is a shift parameter, which accounts for the other factors affecting timber supply than timber price. The value of  $a_w^i$  is calculated for the first period by substituting  $d_w^i$ ,  $\widehat{h}_w^i$ ,  $\widehat{p}_w^i$ , and  $b_w^i$  in equation (11).

The level of the total growing stock volume of timber,  $G_t^i$ , in period  $t$  affects the wood supply tightness in the region via the shift parameter  $a_w^i$ . The growing stock levels in the base year are given as data. Thereafter, the regional growing stock volumes are updated in each period  $t$  employing the specification

$$G_t^i = (1 + g^i) G_{t-1}^i - H_{t-1}^i \quad (12)$$

where  $g^i$  is a growth rate of the growing stock given as data and  $H_{t-1}^i$  is the aggregated harvest of roundwood ( $\sum_w h_w^i$ ) in region  $i$  in period  $t-1$  obtained from the model solution. Assuming unitary inventory elasticity for timber supply, the supply shifter  $\mathbf{a}_w^i$  is updated in each period  $t$  setting

$$\mathbf{a}_{w,t}^i = \frac{\mathbf{a}_{w,t-1}^i}{(G_t^i / G_{t-1}^i)^{b_w^i}} \quad (13)$$

## FOREST INDUSTRY

To each region, a set of alternative production technologies  $l$  for forest industry production are defined. For each technology, input-output coefficients are given that determine the amounts of the inputs used and outputs obtained when one unit of the main product  $m$  of the technology is produced. Besides endogenous sector commodities  $k$  (roundwood, pulp, waste paper, chips, and final products) with input-output coefficients  $a_{kl}$ , there are inputs  $n$  coming from the exogenous sectors, for which the input-output coefficients are denoted by  $a_{nl}$ . These variable inputs  $n$  include net electricity (kWh/unit of main output), process heat (GJ/unit), labor (h/unit), and the aggregate of other exogenous sector inputs, e.g., chemicals and other materials (USD/unit). In addition, maintenance costs for old capacity (USD/unit) and investment costs for new capacity (USD/unit) are accounted for in the production costs.

The prices,  $\mathbf{p}_k^i$ , for the endogenous sector inputs and outputs are determined by the model, while the prices,  $\mathbf{p}_n^i$ , for exogenous, non-forest sector products  $n$  are given as data.

Regarding input-output coefficients, let us follow the sign convention that  $a_{kl} \leq 0$  for inputs,  $a_{kl} \geq 0$  for byproducts and  $a_{ml} = 1$  for the main output. Let the same apply to coefficients  $a_{nl}$ .

Each activity  $l$  in a region  $i$  has a linear production cost function

$$\Gamma_l^i(y_l) = \left( c_l^i - \sum_{k \neq m} \mathbf{p}_k^i a_{kl} \right) y_l^i \quad (14)$$

where  $y_l^i$  is the output of main product  $m$  of the technology and  $c_l^i$  is the unit cost resulting from the exogenous sector inputs  $\sum_n \mathbf{p}_n^i a_{nl}$  and capital cost as discussed below.

A unit investment costs for production capacity of technology  $l$  in region  $i$ ,  $S_l^i$ , is given as data. The capital cost incurring from a unit investment on new production capacity for technology  $l$  in region  $i$  is defined as a certain exogenously specified percentage,  $\mathbf{s}_l^i$ , of the unit investment cost  $S_l^i$  as  $\mathbf{s}_l^i S_l^i$ . This parameter reflects the required return on capital and it may vary across regions due to differences in regional investment risk. For production capacity that has already been taken to use, capital costs have become sunk ( $\mathbf{s}_l^i = 0$ ), but the use of such capacity is subject to maintenance cost that also is assumed to be a certain fraction,  $\mathbf{m}_l^i$ , of the unit investment costs, i.e.,  $\mathbf{m}_l^i S_l^i$ . Hence, for existing capacity,  $c_l^i = \mathbf{m}_l^i S_l^i - \sum_n \mathbf{p}_n^i a_{nl}^i$ , while for a new capacity that is only available through investments,  $c_l^i = \mathbf{s}_l^i S_l^i - \sum_n \mathbf{p}_n^i a_{nl}^i$ . In the next paragraph we discuss the capacity dynamics more closely.

## CAPACITY DYNAMICS IN THE FOREST INDUSTRY

For each production activity  $l$ , a certain region-specific periodical production capacity  $K_l^i$  is defined. For some cases discussed with the model data (Moiseyev et al. 2004), different production technologies may be defined to have a shared production capacity. This takes into account the different input-output mixes that can take place in the mills. A natural example is a sawmill that could produce both hardwood and softwood sawnwood.

In addition to the production technologies and capacities that are available in the base year, investment possibilities exist. These are represented in the model by possible new activities for each product and region, which may be identical to or different from the existing ones regarding their input use. Also for these new production activities  $l$  that are only available through investment maximum periodic-production capacities  $K_l^i$  are defined. Alternatively, the

maximum production level with the new activities can be defined to be shared by several regions. Note that such a capacity  $K_l^i$  for the new activities can be set to be arbitrarily high to avoid constraining investments. (Choices for the capacity limits are discussed with the data in Moiseyev et al. 2004 ).

New production units using any of the new technologies will be taken to use in a region, given that such an investment is considered profitable. Thus, it is required that the market price of the products produced by employing the new technology must cover the per unit variable production costs and the capital costs, i.e.,  $\sum_k a_{kl} p_k^i + \sum_n a_{nl} p_n^i - s_l^i S_l^i \geq 0$  in the market equilibrium. As discussed above, the capital costs resulting from an investment in the new capacity are accounted for only in the period when the installation takes place; thereafter they become sunk costs.

If technology  $l$  is not (or is not likely to be) available in region  $i$ , we set  $K_{lt}^i = 0$ . If the technology related to the new capacity investments is identical to technology already in use in the region, the amount of capacity installed in period  $t$  (=quantity produced with a new technology in period  $t$ ) accumulates the existing capacity with the same input structure in the period  $t+1$  after investment. Else, it remains as a separate existing activity.

While existing capacity may be left idle or divested whenever production with it is unprofitable, a certain fraction of the old capacity can also be specified to be divested exogenously in each period.

Not all capacity increases that are taken place in the real world derive from investments in new production capacity. The forest industry production capacity increases also due to learning or minor regular improvements of the existing capacity. This kind of increase was accounted for in the model by letting the capacity of the low cost technologies to increase by 1 % annually.

## WOOD CONVERSION

In the model, we also have wood conversion activities  $l$  that reflect the fact that some wood categories can be used to substitute others. For instance, softwood sawlogs can be used as softwood pulpwood logs with no extra cost. These conversion activities are defined in the

model in similar terms as the production activities, using conversion coefficients  $a_{wl}$ . Each conversion activity employs one input ( $a_{wl} = -1$ ) and one output ( $a_{wl} = 1$ ). The amount of conversion from one wood category to another in activity  $l$  in region  $i$  is endogenous and denoted by  $y_l^i$ .

### **RECYCLED PAPER SUPPLY**

For each paper grade  $f$  and region  $i$ , we assume that there is a certain maximum percentage (collection rate)  $f_f^i$  of its consumption  $q_f^i$  that can be collected. Collection is subject to a grade specific unit collection cost,  $v_f^i$ , defined as data. These costs set the minimum prices for regional waste paper grades that are eventually endogenously determined in the model. Furthermore, for each  $f$ , we define a set  $\Theta_f$  to consist of those recycled paper categories  $r$  (old newspapers and magazines, mixed grade, high grade etc.) for which collected paper of grade  $f$  is suitable. The model decides endogenously the amount of paper  $f$  collected,  $R_f^i$ , in a region  $i$  and its allocation,  $x_{fr}^i$ , to different waste paper categories  $r$ . Waste paper can be traded with other regions. The cost of processing the waste paper further to pulp are accounted for in the cost components  $c_l^i$  of production activities  $l$  using waste paper  $r$  as an input.

### **INTERREGIONAL TRADE**

For each endogenous sector product  $k$ , a variable cost of transporting one unit of  $k$  per one distance unit has been defined. Then the interregional distance matrix is used to compute the unit variable transportation costs,  $D_k^{ij}$ , as a product of distances and unit variable costs. Alternatively,  $D_k^{ij}$ , can be specified directly.

It is common in trade models to constrain the inter-temporal changes in inter-regional trade. The fact that the trade patterns in real world do not seem to change dramatically from one period to another can be caused by various reasons not captured in the models, e.g., heterogeneity of the products or long-term contracts between the trade partners. Such trade



inertia can be accounted for in the model by setting lower,  $L_k^{ij}$ , or upper  $U_l^{ij}$ , boundaries for trade flow,  $e_k^{ij}$ , of product  $k$  from region  $i$  to  $j$ .

## EXCHANGE RATES

Regional supply and demand functions are specified and updated in the local currencies of the regions, whereas the model is solved in the base currency in the EFI-GTM, which currently is US dollars. The base year exchange rates between local currencies and the US dollar,  $X^{i,0}$ , and the exchange rate projections for the future periods  $t$ ,  $X^{it}$ , are given to the model as data.  $X^{it}$  expresses how many local currency units one unit of the base currency can buy. For instance, in 1999 one US dollar was worth 7.87 Norwegian crowns. Hence,  $X^{it} = 7.87$  for Norway in 1999. For convenience, the exchange rate data may be scaled, e.g., so that  $X^{i0} = 1$  in the base year.

If the rate  $X^{i,t+1}$  of region  $i$  increases from  $X^{i,t}$ , the region's cost competitiveness in US dollar terms improves, but the consumers face higher prices for imported commodities. These relative changes in exchange rates between local currencies and the US dollar are accounted for in the model, and the local costs and demand functions are converted to the US dollar when solving the model for a particular period using the period's exchange rate.

## THE EFI-GTM MODEL SPECIFICATION

Let us now look at how the global problem in Section 2 is modified to adapt these details. We refer to the forest sector products by index  $k$ , of which final products are denoted by  $f$ , pulp grades are denoted by  $u$ , the wood categories are denoted by  $w$ , and the recycled paper categories are denoted by  $r$ . For rest of the notations, the reader is referred to the descriptions above or to the Appendix giving the list of all notations.

The problem below is solved for one period  $t$  at a time. Thereafter the parameters (e.g., parameters in timber supply function, production cost function and demand function) are updated, based on the model solution or external data on the future parameter values.

$$\begin{aligned} \text{Max}_{q_f^i, h_w^i, y_l^i, e_k^{ij}, x_{fr}^i} & \left( \sum_{if} \int_0^{q_f^i} X^{it} P_f^i(q_f) dq_f - \sum_{iw} \int_0^{h_w^i} X^{it} (d_w^i + \mathbf{a}_w^i h_w^i b_w^i) dh_w^i - \right. \\ & \left. \sum_{il} X^{it} c_l^i y_l^i - \sum_{if} (X^{it} v_f^i \sum_{r \in \Theta_f} x_{fr}^i) - \sum_{ijk} X^{it} D_k^{ij} e_k^{ij} \right) \end{aligned} \quad (15)$$

s.t.

$$q_f^i - \sum_l a_{fl} y_l^i + \sum_j (e_f^{ij} - e_f^{ji}) = 0 \quad \forall f, i \quad (16)$$

$$-\sum_l a_{ul} y_l^i + \sum_j (e_u^{ij} - e_u^{ji}) = 0 \quad \forall u, i \quad (17)$$

$$-\sum_l a_{wl} y_l^i - h_w^i + \sum_j (e_w^{ij} - e_w^{ji}) = 0 \quad \forall w, i \quad (18)$$

$$-\sum_{f(r \in \Theta_f)} x_{fr}^i - \sum_l a_{rl} y_l^i + \sum_j (e_r^{ij} - e_r^{ji}) = 0 \quad \forall r, i \quad (19)$$

$$y_l^i \leq K_l^i \quad \forall l, i \quad (20)$$

$$\sum_{r \in \Theta_f} x_{fr}^i \leq \mathbf{f}_f^i q_f^i \quad \forall f, i \quad (21)$$

$$L_k^{ij} \leq e_k^{ij} \leq U_k^{ij} \quad \forall i, j, k. \quad (22)$$

$$e_k^{ij}, q_f^i, y_l^i, h_w^i, x_{fr}^i \geq 0 \quad \forall i, j, k \quad (23)$$

The equations (15)–(23) define a convex optimization problem. Therefore, any solution satisfying the Karush-Kuhn-Tucker conditions of the problem is optimal. It is straightforward to verify that these optimality conditions are in fact equivalent to the conditions of competitive regional equilibrium.

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## APPENDIX: LIST OF INDEXES, PARAMETERS AND VARIABLES

### INDEXES

$t$	time period
$i, j$	regions
$l$	production and conversion activities
$k$	endogenous commodities (forest sector products)
$f$	final forest industry products (products sold to exogenous sectors)
$n$	exogenous sector commodities
$m$	main product of a certain activity
$w$	wood categories
$r$	recycled paper categories
$u$	pulp categories

### PARAMETERS

$\hat{q}_f^i$	reference demand quantity for final product $f$
$\hat{p}_f^i$	reference price for final product $f$
$g_f^i$	price elasticity of the demand for final product $f$
$b_t^i$	forecast of the regional GDP growth for period $t$
$e_f^i$	GDP elasticity of the demand for final product $f$
$\hat{h}_w^i$	reference harvesting quantity for timber category $w$
$\hat{p}_w^i$	reference price for timber category $w$
$d_w^i$	exogenous component in the timber price that does not depend on harvest quantity
$b_w^i$	inverse of the price elasticity of timber supply for timber category $w$
$a_w^i$	shift parameter in timber supply function (calculated from the other data)
$G_t^i$	total growing stock volume of forests (specified for the base year, updated automatically)
$g^i$	growth rate of the growing stock of forests
$a_{kl}$	input ( $a_{kl} \leq 0$ ) or output ( $a_{kl} \geq 0$ ) of endogenous commodity $k$ in production or conversion activity $l$ per one output unit of the main product $m$ ( $a_{ml} = 1$ ) of the activity

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$a_{nl}$	input or output of exogenous commodity $n$ in production activity $l$ per one output unit of the main output $m$ of the activity
$p_n^i$	price for exogenous sector commodity $n$
$S_l^i$	a unit investment costs for production capacity of technology $l$ in region $i$
$s_l^i$	percentage of the total unit investment cost $S_l^i$ that has to be covered (applicable only in the period the investment takes place, $s_l^i = 0$ for existing production activities).
$m_l^i$	variable maintenance costs as a fraction of unit investment costs
$c_l^i$	unit variable costs resulting from the use of exogenous sector inputs in technology $l$ (calculated from the other data)
$K_l^i$	maximum production level (capacity) for activity $l$ (Additionally, common maximum production levels may be defined for several production activities and/or for activities in several regions).
$f_f^i$	a maximum percentage of the consumption $q_f^i$ of paper product $f$ that can be collected (collection rate)
$v_f^i$	a unit collection cost of paper grade $f$
$\Theta_f$	a set including recycled paper categories $r$ in which paper $f$ can be allocated
$D_k^{ij}$	unit transportation cost for product $k$ from $i$ to $j$
$L_k^{ij}$	minimum export quantity for product $k$ from $i$ to $j$
$U_l^{ij}$	maximum export quantity for product $k$ from $i$ to $j$
$X^{it}$	exchange rate of the region's $i$ currency in period $t$ with respect to the base currency (us\$)

Note that while time period  $t$  is omitted from the symbols above, any piece of the above parametric data can be defined to be time specific. E.g., prices for exogenous commodities may be defined for each period differently or trade inertia boundaries may be defined to depend on the previous periods solution.

**VARIABLES**

For each period  $t$ , the model calculates the equilibrium values for the following variables in regions  $i$ :

$q_f^i$	consumption of final product $f$
$y_l^i$	output of main commodity $m$ of activity $l$
$h_w^i$	harvested quantity of roundwood $w$
$e_k^{ij}$	export quantity for product $k$ to from region $i$ to $j$
$x_{fr}^i$	amount of collected waste paper $f$ that is graded to category $r$
$p_k^i$	prices for endogenous sector products $k$

## THE EFI-GTM DATA REQUIREMENTS AND THE DATA SOURCES CURRENTLY USED

	Description (see <i>parameters</i> above)	Data sources
$\widehat{q}_f^i$	reference demand quantities for final products	Jaakko Pöyry Consulting (JPC)
$\widehat{p}_f^i$	reference prices for final products	JPC
$g_f^i$	price elasticities of the demand for final products	Based on FAO (1997) & FAO (1999) and authors judgement
$b_t^i$	GDP growth forecasts	IMF World Economic Outlook Database, April 2003 ( <a href="http://www.imf.org/external/pubs/ft/weo/2003/01/data/growth_a.csv">http://www.imf.org/external/pubs/ft/weo/2003/01/data/growth_a.csv</a> ) estimates for 2000–2004, 2005–2020 by authors
$e_f^i$	GDP elasticities for final products	Based on FAO (1997) & FAO (1999) and authors judgement
$\widehat{h}_w^i$	reference timber harvests	FAOSTAT database, forestry data( <a href="http://faostat.fao.org/faostat/collections?subset=forestry">http://faostat.fao.org/faostat/collections?subset=forestry</a> ), and JPC
$\widehat{p}_w^i$	reference timber prices	JPC
$\frac{1}{b_w^i}$	timber price elasticities	Assumed at 0.5 for EU15 countries, 1.3 for Russia and 1 for other regions based on previous existing econometric country studies
$d_w^i$	exogenous part in timber price	Set at zero
$G_0^i$	base year forest growing stock volumes	UNECE/FAO (2000) for Europe, North America, Australia, Japan and New Zealand, FAO (1998) for other regions
$g^i$	forest growing stock growth rates	
$a_{kl}$	input coefficients for technologies	JPC
$p_n^i$	prices for exogenous sector products	JPC for the base year, assumed to stay at the base year level thereafter
$S_l^i$	unit capacity investment costs for production technologies	JPC

$S_l^i$	percentage of unit investment costs that has to be covered in order to use new capacity	Assumed to be in the range of 15–25% (15% for developed industrialised regions, 25% for regions with highest investment risk, where country investment risk is based on ranking by FINNVERA ( <a href="http://www.finnvera.fi/">http://www.finnvera.fi/</a> ))
$m_l^i$	variable maintenance costs	Assumed to be 5% of the unit investment cost
$K_l^i$	production capacities	JPC for production activities existing in the base year, for new production activities, see Moiseyev et al. (2004)
$f_f^i$	maximum paper collection rates	Based on FAOSTAT recovered paper production and trade data ( <a href="http://faostat.fao.org/faostat/collections?subset=forestry">http://faostat.fao.org/faostat/collections?subset=forestry</a> )
$v_f^i$	unit paper collection costs	JPC
$D_k^{ij}$	unit transportation costs	JPC
$L_k^{ij}$	minimum export quantities	Set at zero
$U_l^{ij}$	maximum export quantities	No limit (= not used)
$X^{it}$	exchange rates	Assumed to stay at the base year level in all the periods